

# Sense and Sensitivity: An input space odyssey for ABS Ratings

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## Abstract

Asset backed securities (ABSs) are structured finance products backed by pools of assets and are created through a securitization process. The assessment of asset backed securities is given by ratings partly based on a quantitative model for the defaults and prepayments of the assets in the pool. This mathematical approach contains a number of assumptions and estimations of input variables whose values are affected by uncertainty. The uncertainty in these variables propagates through the model and produces uncertainty in the ratings. In the present paper we propose to work with global sensitivity analysis techniques to investigate ABS ratings sensitivity to the input parameters and we introduce a novel structured financial rating to take into account uncertainty in assessment when rating ABSs.

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# 1 INTRODUCTION

An asset-backed security (ABS) is a security created through a securitization process whose value and income payments are backed by a specific pool of underlying assets. Illiquid assets that can not be sold individually (e.g. mortgages, car loans, and SME loans) are pooled together and transferred to a shell entity specially created to be bankruptcy remote (a so called Special Purpose Vehicle (SPV)) which in turn issues notes (liabilities) to investors with distinct risk return profiles and different maturities. This process is called securitization.

The valuation of securitization transactions is given by ratings addressing the different risks inherent in a structure and how well these risks are mitigated. The rating process is partly based on quantitative analysis of how the transaction mitigates default and prepayment scenarios. These scenarios are generated by more or less sophisticated models with one or more parameters. Typically the input parameters are unknown and estimated from historical data or given by expert opinions. In any way, the values used for the parameters are uncertain and these uncertainties are propagated through the model and generates uncertainty in the rating output. For an introduction to ABS and the risks and the rating methodology see Jönsson and Schoutens (2009), Jönsson and Schoutens (2010), and Jönsson et al. (2009).

There have been an increased attention to the rating of asset backed securities during the credit crisis 2007–2008 due to the enormous losses anticipated by investors and the huge amount of downgrades among structured finance products. Rating agency have been encouraged to sharpen their methodologies and to provide more clarity to the limitations of their ratings and the sensitivity of those ratings to the risk factors accounted for in their rating methodologies (see, e.g., Moody's Investor Service (2000), Moody's Investor Service (2009), and IMF Global Stability Report, April 2008, p. 81). Moody's in Moody's Investor Service (2000) has introduced the volatility scores (V Scores). They are a technique based on a qualitative analysis to assess the quality of available credit information and the potential variability around various inputs to a rating determination. Moody's in Moody's Investor Service (2009) provides a calculation of the number of rating notches that a Moody's rated structured finance security may vary if certain input parameters differed. Here two input parameters, the mean portfolio default rate and the mean recovery rate, have been stressed by using several combinations of possible values. This is a quantitative analysis that highlights how changes in the two input parameters, mean and recovery rate, can affect the rating determination. In IMF Global Stability Report (April 2008, p. 82), it has also been suggested that a rating scale different than the one used for corporate and sovereign bonds should be used.

Starting from these considerations, the objectives of this paper are two fold. Firstly, we advocate the use of techniques to enhance the understanding of the variability of the ratings due to the uncertainty in the input parameters used. Uncertainty analysis assesses and quantifies the variability in the output of interest due to the variability in the inputs. Uncertainty analysis can be paired with a global sensitivity analysis, which is used to understand the main sources of output uncertainties and how the uncertainty in the output can be allocated to the different sources of uncertainty in the inputs. We quantify the percentage of output variance that each input factor is accounting for and we also detect how interactions among input parameters affect the rating variability exploring the whole input space (see Saltelli et al. (2008) and Saltelli et al. (2004)). The idea is to answer to the following questions: Is the rating of an ABS reliable? Where does the uncertainty come from, i.e. which input factors are more important in determining the uncertainty in the rating response? Can I quantify the exact percentage of the variability in the output that can be allocated to each input?

Secondly, we propose a novel rating approach called *global rating*, that takes this uncertainty

in the output into account when assigning ratings to tranches. The global ratings should therefore become more stable and reduce the risk of cliff effects, that is, that a small change in one or several of the input assumptions generates a dramatic change of the rating. The global rating methodology proposed gives one answer of a way forward for the rating of structure finance products.

The rest of the paper is outline as follows. In the next section we give an introduction to ABS, we describe the basic steps of modelling the cash flows produces by the asset pool, we point out the collection of the cash flows and the distribution of these cash flows to the liabilities and we outline the procedure to get ratings.

A description of general elements of SA is provided in Section 3 with a particular attention to the techniques used in this paper.

Section 4 analyses in depth the ratings to improve the assessment of the risk embedded in ABSs structure. Uncertainty analysis is used to detect the uncertainty inside the model and sensitivity analysis techniques are applied to determine the responsible of it. An attempt to take into account this uncertainty when rating ABSs is proposed in Section 5. The paper ends with a conclusion.

## 2 ASSET-BACKED SECURITIES

Asset-backed securities (ABSs) are securities created through a securitization process whose value and income payments are backed by a specific pool of underlying assets (see Fabozzi and Kothari (2008)). Illiquid assets cannot be sold individually so that they are pooled together by the originator (Issuer) and transferred to a shell entity specially created to be bankruptcy remote, (a so called Special Purpose Vehicle (SPV)) which in turn issues notes (liabilities) to investors with distinct risk return profiles and different maturities: senior, mezzanine, and junior notes. This technique is called tranching of the liability. Cash flows generated by the underlying assets are used to service the notes; the risk of the underlying assets results to be diversified because each security now is representing a fraction of the total pool value. Figure 1 shows a general ABS structure.

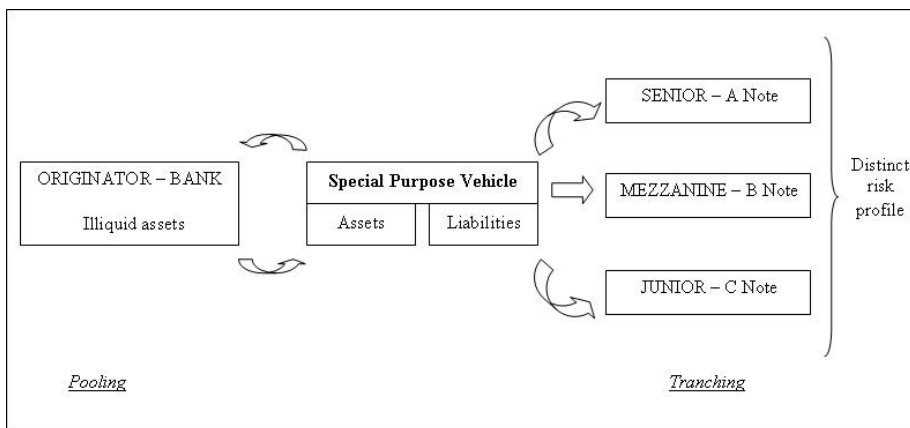


Figure 1: ABS general structure.

The assessment of the ABS is related with the risks inherent in the structure. The ratings are indicators of the credit risk embedded in these instruments. To derive a final rating of asset-backed securities, rating agencies combine both a qualitative assessment and quantitative analysis, which assess the originator’s and the servicer’s operations and legal issues concerning the transfer of the assets from the originator to the issuer (Moody’s Investor Service (2001), Moody’s Investor Service (2007a) and Standard and Poor’s (2007)).

The qualitative assessment is described in detail in the deal’s prospectus and it is mostly a matter of translating legal descriptions of how the available funds should be distributed among different stake holders (issuer, servicer, ABS investors, counterparties, etc.). The quantitative analysis relies on modelling of the cash flows produced by the assets (based on default and prepayment models of different level of sophistication), the collections of these cash flows and the distribution of the cash flows to the liabilities according to a payment priority (waterfall).

In this section, we introduce the ABS structure we are going to use in the numerical experiment, we describe the basic steps of modelling the cash flows produced by the assets in the pool (default models), we point out the collection of the cash flows and the distribution of these cash flows to the liabilities and we outline the rating of asset-backed securities.

## 2.1 The ABS structure for the experiment

Through out the paper we assume that the pool is homogeneous, i.e., that all the constituents of the pool are identical with respect to initial amount, maturity, coupon, amortisation and payment frequency, see Table 1, and with respect to risk profile (i.e. probability of default). This implies that the pool is assumed behave as the average of the assets in the pool. We also assume the pool to be static, i.e. no replenishment is done.

Collateral	
Number of loans	2,000
Initial principal amount	100,000,000
Weighted average maturity	5 years
Weighted average coupon (per annum)	9%
Amortisation	Level-Pay
Payment frequency	Monthly

Table 1: Collateral characteristics.

This collateral pool is backing three classes of notes: A (senior), B (mezzanine), and C (junior). The details of the notes are given in Table 2 together with other structural characteristics. To this basic liability structure we have added a cash reserve account.

Liabilities			
Class of Notes	Initial Principal Amount	Interest Rate (per annum)	Credit enhancement (%)
A	80,000,000	1%	20%
B	14,000,000	2%	6%
C	6,000,000	4%	0%
General Features			
Final Maturity		10 years	
Payment frequency		Monthly	
Principal allocation		Sequential	
Shortfall rate (per annum)		Applicable note coupon	
Senior expenses			
Issuer fees		1% of Outstanding Pool Balance	
Servicer fees		1% of Outstanding Pool Balance	
Payment frequency		Monthly	
Shortfall rate (per annum)		20%	
Cash reserve			
Target amount		1% of Outstanding Pool Balance	
Minimum required amount		0% of Outstanding Pool Balance	

Table 2: Liability and structural characteristics.

The priority of payments of the structure, the waterfall, is presented in Table 3. The waterfall is a so called combined waterfall where the available funds at each payment date constitutes of both interest and principal collections.

Waterfall	
Level	Basic amortisation
1)	Issuer expenses
2)	Servicer expenses
3)	Class A interest
4)	Class B interest
5)	Class A principal
6)	Class B principal
7)	Reserve account reimburs.
8)	Class C interest
9)	Class C principal
10)	Class C additional returns

Table 3: The waterfall used in the analysis.

The assessment of risk embedded in this ABS structure is given by ratings based on quantitative assessment of the cash flow produced by the assets and how these are distributed to the liability.

## 2.2 Cash flow modelling

The modelling of the cash flows produced by the assets in the pool is based on default and prepayment models. To simplify the experiment the loans are assumed to be able to default but not prepay. The approach followed by Moody's to analyze Small Medium Enterprise (SME) transactions depends on the size and granularity of the underlying portfolio (Moody's Investor Service (2007b)).

If the portfolio is non granular, the defaults can be generated by factor models, typically the Gaussian one-factor copula model or its multi-factor version. In this case the defaults are generated by simulating the individual assets defaults, combined with stochastic recovery rates. See, for example, Moody's CDOROM<sup>TM</sup>.

For a granular portfolio, as the one in our experiment, the default scenarios can be generated by slicing a default distribution in thin slice (see Figure 2), each slice representing a cumulative default rate scenario. The probability of each scenario is given by the default distribution.

These cumulative default rate scenarios are then distributed over the life of the pool using one or several default timing vectors, preferably estimated from historical default patterns (see Jönsson and Schoutens (2009)).

Instead of slicing the distribution we use Monte Carlo simulations that have the benefit of giving error estimates of the output in the form of confidence intervals. Different default scenarios are generated by first sampling a cumulative portfolio default rate from a default distribution and then distribute this default rate over time with the help of a default curve. The **default distribution** of the pool is assumed to follow a Normal Inverse distribution in accordance with Moody's methodology and the **default curve** is modelled by the Logistic model. These distributions are characterised by input parameters which have to be given by expert opinions or estimated from historical data on the performance on the asset pools with similar characteristics as the asset pool under consideration. Moreover if the asset pool is revolving, that is, new assets are added to the pool when existing assets are removed from the pool due to repayments or defaults, the uncertainty of the pool performance is even more uncertain. Thus, the quantitative

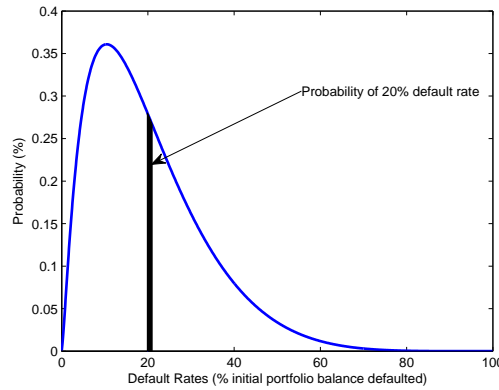


Figure 2: Illustration of a default distribution. The 20% Default Scenario and its associated probability is marked with a bar.

analysis introduces an exposure to parameter uncertainty. Under the assumption to set up the default models by using these two distributions we do not have to take into account the model uncertainty and we let just the uncertainty in the parameters which have to be fixed at the beginning. The impact of model choice has been presented in Jönsson and Schoutens (2010) and Jönsson et al. (2009).

Recovery rate and recovery timing assumptions have to be added to each default scenario. See, for example, Moody's Investor Service (2006)).

### The default distribution - Normal Inverse

Let  $PDR(T)$  denote the portfolio default rate at time  $T$  of our large homogeneous portfolio. The distribution of  $PDR(T)$  is given by the the following Normal Inverse:

$$F_{PDR(T)}(y) = P[PDR(T) < y] = \Phi \left( \frac{\sqrt{1-\rho}\Phi^{-1}(y) - \Phi^{-1}(p(T))}{\sqrt{\rho}} \right) \quad (1)$$

where  $0\% \leq y \leq 100\%$ ,  $\rho$  is the obligor correlation and  $p(T)$  is the probability of default by  $T$  of a single obligor in the pool.

This Normal Inverse distribution to generate default scenarios is derived as an approximation to the distribution of the portfolio default rate at maturity  $T$  when the Gaussian one-factor model is used to model the defaults in a large homogeneous portfolio where the number of assets in the pool is assumed to grow to infinity.

The default distribution in (1) is a function of the obligor correlation,  $\rho$ , and the default probability,  $p(T)$ , which are unknown and unobservable. Instead of using these parameters as inputs it is common to fit the mean and standard deviation of the distribution to the mean and standard deviation, respectively, estimated from historical data (see, for example, Moody's Investor Service (2007b) and Raynes and Rutledge (2003)). Let us denote by  $\mu_{cd}$  and  $\sigma_{cd}$  the estimated mean and standard deviation, respectively.

The mean of the distribution is equal to the probability of default for a single obligor,  $p(T)$ , so  $p(T) = \mu_{cd}$ . As a result there is only one free parameter, the correlation  $\rho$ , left to adjust to fit the distribution's standard deviation to  $\sigma_{cd}$ , which can be done numerically by minimizing

$\sigma_{cd} - Var_{\rho}(PDR(T))$ , where the subscript in  $Var_{\rho}$  is used to show that the variance is a function of  $\rho$ . Figure 3 shows some typical examples of Normal Inverse distributions.

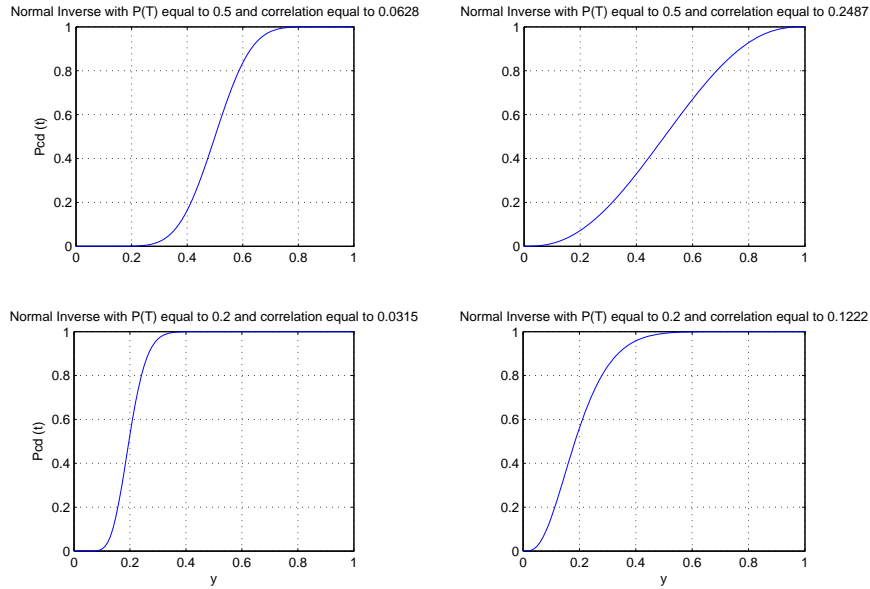


Figure 3: Portfolio default rate for different values of  $\rho$  and/or  $p(T)$ .

### The default curve - Logistic Function

The default curve represents the cumulative portfolio default rate evolution over time and it is thus used to distribute the default in the pool over time. It provides the percentage of the total cumulative default rate that will be applicable in each month. The curve should therefore be monotonically increasing and the slope of the curve should be always non negative.

The function used to model the default timing will be a common sigmoid curve: the Logistic Function (see Figure 4). The name has been given in 1838 by Pierre François Verhulst who modelled the **S-shaped curve** of growth of some populations with this function (see Richards (1959), and Verhulst (1838)). This function is the among the simplest non linear curves and it finds application in a range of fields including biology, sociology, economics, probability, and statistics. In its most basic form, the logistic dynamic has been expressed by the following Ordinary Differential Equation (ODE):

$$\frac{dF(t)}{dt} = c \left( 1 - \frac{F(t)}{a} \right) P(t), \tag{2}$$

and it is easy to derive the solution:

$$F(t) = \frac{a}{1 + \left(\frac{a}{F_0} - 1\right)e^{-c(t-t_0)}}.$$

In order to simplify the expression and without loss of generality,  $\left(\frac{a}{F_0} - 1\right)$  will be considered just as one parameter,  $b$ , such that the expression can be rewritten as follows:



$$F(t) = \frac{a}{1 + be^{-c(t-t_0)}}, \quad 0 \leq t \leq T, \quad (3)$$

where  $a$ ,  $b$ ,  $c$ , and  $t_0$  are positive constants. Parameter  $a$  controls the right endpoint of the curve. In fact, for large values of  $T$  in the ABS model,  $a$  can be considered a good approximation of the cumulative default rate of the pool at maturity since  $\lim_{t \rightarrow +\infty} F(t) = a$ . Thus, we can easily use the Logistic function in a Monte Carlo based scenario generator. All we have to do is to generate stochastic default scenario by sampling the cumulative portfolio default rate,  $a$ , from the Normal Inverse distribution. Due to the high computational cost of the ABS model, we use Quasi-Monte Carlo approach based on Sobol sequences to get a faster convergence of our model. Sobol sequences, in fact, are called quasi-random sequences and they generate a sample of points as uniformly as possible with the advantage of faster convergence after relative few runs in comparison with Monte Carlo<sup>1</sup> (See Kucherenko (2008), Kucherenko et al. (2010), Kucherenko (2007), and Kucherenko et al. (2000).) By using this approach, we speed up the simulation and we get the model convergence with just  $2^{14}$  sampled cumulative default rate,  $a$ . They have been generated from the normal inverse distribution by using Sobol sequences and we run our model for each of them.

Parameter  $b$  is a curve adjustment factor. Early increase in the default rate can be achieved by setting  $b$  to a low value. Increasing  $b$  means delaying the steeping of the default curve. Parameter  $c$  is a spreading factor determining how spread out the curve is around  $t_0$ . It can be thought of as the standard deviation of the default curve. Parameter  $t_0$  is the inflection point of the model:  $F(t)$  grows at an increasing rate before  $t_0$  and at a decreasing rate afterwards. It is controlling the time point of the maximum marginal defaults. If  $b = 1$ , the curve becomes symmetric around  $t_0$ .

The shape of the Logistic function and the influence of the parameters are illustrated in Figure 4.

Note that  $F(0) \neq 0$ , so in order to have no defaults in the pool at inception we must subtract  $F(0)$  from  $F(t)$ , for all  $t$ . We also want  $a$  to be the cumulative portfolio default rate at maturity, i.e.  $F(T) - F(0) = a$ . Thus the normalized Logistic function we will use as default model is:

$$\hat{F}(t) = (F(t) - F(0)) \frac{a}{F(T) - F(0)}, \quad t \geq 0.$$

The monthly fraction of defaults at month  $t_m$  is given by

$$\hat{p}(t_m) = \hat{F}(t_m) - \hat{F}(t_{m-1}), \quad m = 1, 2, \dots, M,$$

and the cumulative default rate at time  $t_m$

$$\sum_{i=1}^m \hat{p}(t_i) = \hat{F}(t_m), \quad m = 1, 2, \dots, M.$$

### 2.3 Collection and distribution of the cash flows

After generating defaults in the pool under a fixed set of input parameters, we point out here briefly the monthly collection and distribution of the cashflows.

We denote by  $t_m$ ,  $m = 0, 1, \dots, M$  the payment date at the end of month  $m$ , with  $t_0 = 0$  being the closing date of the deal and  $t_M = T$  being the final legal maturity date.

<sup>1</sup>The efficiency of MC methods is determined by the proprieties of the random numbers. It is know that random sampling is prone to clustering: for any sampling there are always empty areas as well as regions in which random points are wasted due to clustering.

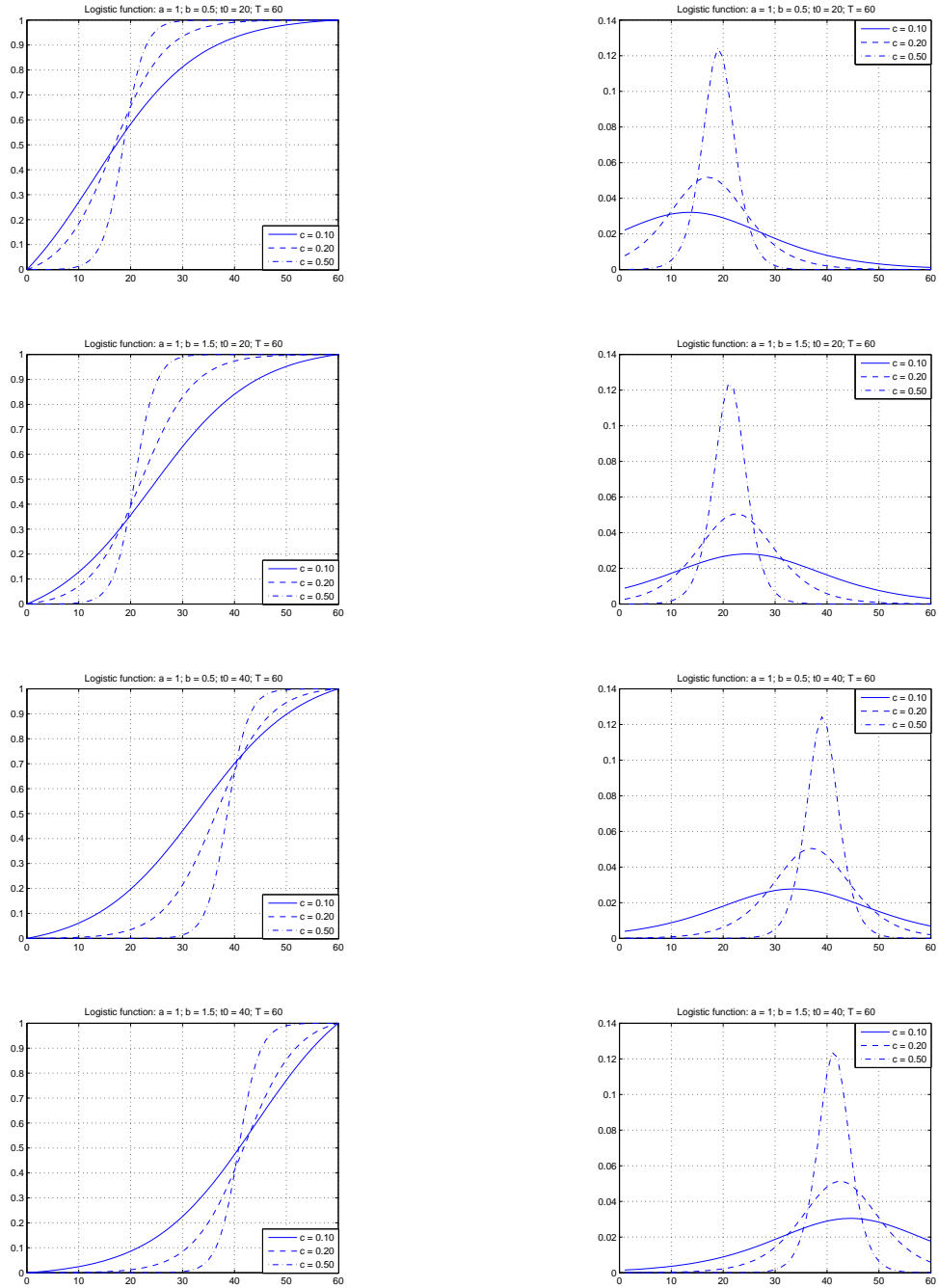


Figure 4: The Logistic function and its derivative for different values of  $b$ ,  $c$ , and  $t_0$ . Parameter values:  $a = 1$  and  $T = 60$ .

The cash collections each month from the asset pool consists of interest payments and principal collections (scheduled repayments) and together with the principal balance of the reserve account constitute available funds.

### Asset behavior

We begin by modelling the asset behavior for the current month, say  $m$ . The number of performing loans in the pool at the end of month  $m$  will be denoted by  $N(t_m)$ . We denote by  $n_D(t_m)$  the number of defaulted loans in month  $m$  coming from the Monte Carlo simulation. Note that

$$N(t_m) = N(t_{m-1}) - n_D(t_m).$$

### Default principal

Defaulted principal is based on previous months ending principal balance times number of defaulted loans in current month:

$$P_D(t_m) = B(t_{m-1}) \cdot n_D(t_m),$$

where  $B(t_m)$  is the outstanding principal amount at time  $t_m$  of an individual loan and  $B(0)$  is the initial outstanding principal amount.

### Interest collections

Interest collected in month  $m$  is calculated on performing loans, i.e., previous months ending number of loans less defaulted loans in current month:

$$I(t_m) = (N(t_{m-1}) - n_D(t_m)) \cdot B(t_m) \cdot r_L,$$

where  $N(0)$  is the initial number of loans in the portfolio and  $r_L$  is the loan interest rate. It is assumed that defaulted loans pay neither interest nor principal.

### Principal collections

Scheduled repayments are based on the performing loans from the end of previous month less defaulted loans:

$$P_{SR}(t_m) = (N(t_{m-1}) - n_D(t_m)) \cdot B_A(t_m),$$

where  $B_A(t_m)$  is scheduled principal amount paid from one single loan.

### Recoveries

We will recover a fraction of the defaulted principal after a time lag,  $T_{RL}$ , the recovery lag:

$$P_{Rec}(t_m) = P_D(t_m - T_{RL}) \cdot RR(t_m - T_{RL}),$$

where  $RR$  is the Recovery Rate.

### Available Funds

The available funds in each month, assuming that total principal balance of the cash reserve account ( $B_{CR}$ ) is added, is:

$$I(t_m) + P_{SR}(t_m) + P_{Rec}(t_m) + B_{CR}(t_m).$$

## Total principal reduction

The total outstanding principal amount on the asset pool has decreased with:

$$P_{Red}(t_m) = P_D(t_m) + P_{SR}(t_m),$$

and to make sure that the notes remain fully collateralised we have to reduce the outstanding principal amount of the notes with the same amount.

After collecting the cash flow, the allocation of principal due to be paid to the notes is supposed to be done sequentially, which means that principal due is allocated in order of seniority according to the waterfall (see Table 3). In the beginning, principal due is allocated to the Class A notes. Until the Class A notes has been fully redeemed no principal is paid out to the the other classes of notes. After the Class A notes are fully redeemed, the Class B notes are started to be redeemed, and so on.<sup>2</sup> Note that we here are discussing the calculation of principal due to be paid. The actual amount of principal paid to the different notes depends on the available funds at the relevant level of the waterfall.

## 2.4 Ratings of ABSs

In order to evaluate an ABS under a fixed set of input parameters, we need to come to a rating output that addresses either the **Expected Loss** an investor might incur or the **probability of default**. The probability of default approach assesses the likelihood of full and timely payment of interest and the ultimate payment of principal no later than the final legal maturity. The expected loss approach is an assessment of the default probability and the loss severity given default. In the sequel, we will use the expected loss approach.

After the collection and distribution of the cash flows as pointed out in the Section 2.3, we evaluate the relative net present value loss and the weighted average life of each class of notes in each default scenario simulation. The relative net present value loss for a note is calculated by discounting the cashflows (both interest and principal) received on that note and by comparing it to the initial outstanding amount on the note (Moody's Investor Service (2006)):

$$\text{Relative PV Loss(Scenario } s) = \frac{\text{Nominal Initial Amount} - \text{PV Cashflow(Scenario } s)}{\text{Nominal Initial Amount}}.$$

For a fixed rate note the discount rate will be the promised coupon rate. The weighted average life for a note is calculated as follows:

$$\sum_{m=1}^M \frac{\text{Outstanding Note Amount}(\text{Time } t_m, \text{Scenario } s)}{\text{Original Note Amount} \times 12},$$

where month  $M$  is the month ending with the legal maturity date  $t_M$ .

Under the assumption to generate  $2^{14}$  cumulative default rate to get the convergence of our model, after evaluating each scenario we calculate one Expected Loss and one Expected Average Life. The Expected Loss is the average loss over all the scenarios. It is calculated by averaging the Relative Present Value Loss representing the percentage loss in each scenario.

The Expected Loss is then given by:

$$\text{Expected Loss} = \frac{1}{\text{Max Num. Scenarios}} \sum_{s=1^{\text{st Default Scenario}}}^{\text{Max Num. Scenarios}} \text{Relative PV Loss(Scenario } s).$$

<sup>2</sup>There are two ways to allocated principal due: sequential and pro rata. In a pro rata scheme principal due is allocated proportionally to the outstanding principal balance of the notes.

The Expected Average Life (or expected weighted average life) of the note (in years) is the weighted average of the times of the principal payments. It is the average time until a dollar of principal is repaid. It is calculated as follows:

$$\frac{1}{\text{Max Num. Scenarios}} \sum_{s=1\text{st Default Scenario}}^{\text{Max Num. Scenarios}} \text{Weighted Average Life(Scenario } s),$$

The rating of a note is found from Moody's Idealised Cumulative Expected Loss Table, which maps the Expected Loss and the Expected Average Life combination to a specific quantitative rating. An example of such a table is given in Moody's Investor Service (2000).

Summarizing, under the assumption that the setting of the input parameters has been fixed, at the beginning we generate  $2^{14}$  cumulative default rates,  $a$  values, from the normal inverse distribution and we run the model for each of them. Following, by using an average over all these scenarios we come up to a single rating which provides one evaluation of ABSs model associated to the fixed setting of input parameters.

### 3 GLOBAL SENSITIVITY ANALYSIS

We have already seen that the assessment of the ABSs is based on a quantitative model, already explained, containing some input parameters whose values are affected by uncertainty. This uncertainty propagates through the model and generates uncertainty in the rating output. We are going to fill the need of investigating the rating sensitivity with respect to input assumptions with the help of sophisticated methods. By using global sensitivity analysis<sup>3</sup> (SA), we want to investigate on this uncertainty. In general terms, in fact, global sensitivity analysis is the study of how the uncertainty in a model input affects the model's response and investigates the different sources of uncertainty in the model inputs. Different sensitivity analysis techniques can be followed to test the sensitivity of a model, ranging from global variance method (see Saltelli (2002), Saltelli et al. (2008) and Saltelli et al. (2004)), which decomposes quantitatively the total output variance into contributions of each input, to the simplest class of the screening tests which provides a qualitative information by varying one factor at a time.

The rating procedure described so far is based on the assumption that all the input values are fixed at the beginning and we get one evaluation of the ABSs model associated to this setting of parameters providing one single rating for each tranche. Now, the start point of the sensitivity analysis is to evaluate ABSs model several times by using several settings of input parameters in order to take into account that each input can assume a discrete number of values within their ranges: we generate several values for the input factors and for each of them we evaluate the model as has been explained in Section 2. The first class requires a high number of SA model evaluations and an extreme computational cost but we take advantage of using it because we get the contribution of each input factor to the variance of the output taking into account the interactions among factor. Within the screening methods, elementary effects method (EE method) identifies important factors with few SA simulations. It is very simple and easy to implement and the results are clear to be interpreted. It has been introduced by Morris (1991) and has been refined by Campolongo and co-workers in Campolongo et al. (2007).

Because of the ABSs structure's complexity, our model is computationally expensive and the EE method is very well suited to screen the input space in a first step. All the not influential factors will be determined and their values will be fixed without affecting the output variance of interest. Following, the variance based method will be applied to quantify and to distribute the uncertainty of our model among the parameters identified to be influential by the elementary effect.

#### 3.1 Elementary Effects

The **Elementary Effect** (EE) of a specific input factor is the difference in the model output when this particular input factor is changed, while the rest of the input factors are kept constant. The method is thus based on one-at-a-time sensitivity analysis. However, in the EE method the one-at-a-time analysis is done many times for each input, each time under different settings of the other input factors, and the sensitivity measures are calculated from the empirical distribution of the elementary effects.

Let us assume that there are  $k$  uncertain input parameters  $X_1, X_2, \dots$ , and  $X_k$  (assumed to be independent) in our model. Examples of input parameters are the mean and standard

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<sup>3</sup>In the literature, most of the sensitivity analysis is based on derivatives. It is the so called 'local sensitivity analysis' where the factor' importance is investigated by derivatives of the output with respect to that factor. The term 'local' refers to the fact that all the derivatives are taken at a single point in the input space. Local techniques cannot be used for the robustness of model unless the model is proven to be linear or additive.

deviation of the default distribution.

To each input factor we assign a range and a distribution. For example, we could assume that  $X_1$  is the mean of the default distribution and that it takes values in the range  $[5\%, 30\%]$  uniformly, that is, each of the values in the range is equally likely to be chosen. We could of course use non-uniform distributions as well, for example, an empirical distribution.

These input parameters and their ranges create an input space of all possible combinations of values for the input parameters. To apply the EE method we map each of the ranges to the unit interval  $[0, 1]$  such that the input space is completely described by a  $k$ -dimensional unit cube.

In order to estimate the sensitivity measure which is able to detect input factors with an important overall influence on the output, a number of elementary effects must be calculated for each input factor. We build  $r$  trajectories in order to compute  $r$  elementary effects (see Morris (1991)). Each trajectory is composed by  $(k + 1)$  points in the input space such that each input factor changes value only once of a step equal to  $\Delta$ . A characteristic of this design is that the points on the same trajectory are not independent and in fact two consecutive points differ only in one component. Points belonging to different trajectories are independent since the starting points of the trajectories are independent.

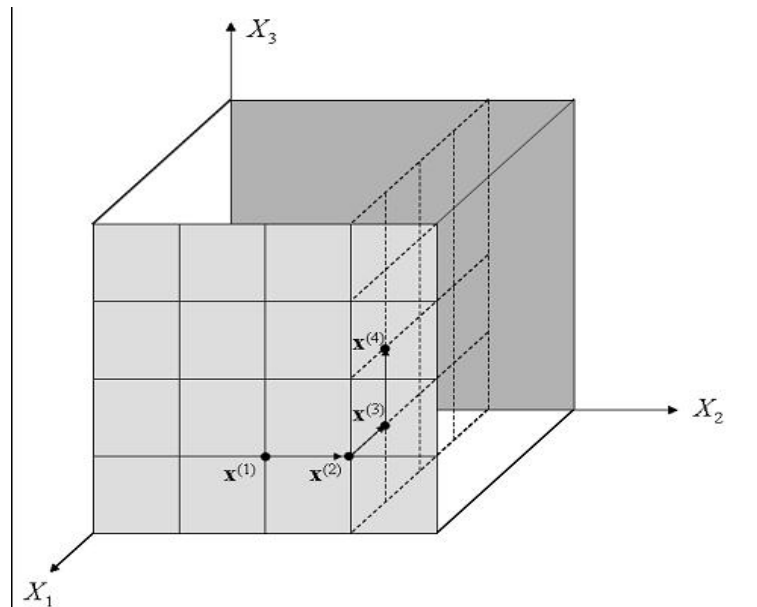


Figure 5: Trajectory in the input space.

Once a trajectory has been generated, the model is evaluated at each point of the trajectory and one elementary effect for each input factor can be computed.

The EE of input factor  $i$  is as follows:

$$|EE_i(\mathbf{X}^{(l)})| = \frac{|Y(\mathbf{X}^{(l+1)}) - Y(\mathbf{X}^{(l)})|}{\Delta} \quad (4)$$

By randomly sampling  $r$  trajectories,  $r$  elementary effects can be estimated for each input. Usually the number of trajectories,  $r$ , depends on the number of factors and on the computational cost of the model and it has been proven that the best choice is to use 10 trajectories (see Saltelli et al. (2008), and Saltelli et al. (2004)). See Campolongo et al. (2007), for the all details about the design that builds the  $r$  trajectories of  $(k + 1)$  points in the input space.

Starting from the absolute values of the elementary effects, we introduce the sensitivity measure of each input that can be used to assess the importance of each factor in the model:

$$\mu_i^* = \frac{\sum_{j=1}^r |EE_i^j|}{r}. \quad (5)$$

Higher  $\mu_i^*$  value, more important factor  $i$ .

### 3.2 Variance based method

Partitioning the output variance of the model gives a way of performing sensitivity analysis. The idea is the decomposition of the variance into contributions of each input factor.

Let us assume that there are  $k$  uncertain input parameters  $X_1, X_2, \dots$ , and  $X_k$  in our model and to each input factor we assign a range and a distribution. For example, we could assume that  $X_1$  is the mean of the default distribution and that it takes values in the range [5%, 30%]. If we knew the value for the input  $X_1$  (for example to be equal to  $x_1^*$ ), we could calculate the conditional variance:

$$V(Y|X_i = x_i^*)$$

and it will be a good sensitivity measure.

The point is that the real value of an input factor is unknown and each input factor can assume a discrete number of values through its range. In order to take into account that each input factor has distributed accordingly to some distribution, the model is run different times by varying the factors among a set of values. To generate the set of input values, we map the ranges to the unit interval [0, 1] and all the values are sampled quasi random by using Sobol' sequences. By averaging the conditional variance over all possible values for the factor  $X_i$ , we overcome the problem of do not know the real value for the input  $X_i$ :

$$E_{X_i}(V_{X_{-i}}(Y|X_i))$$

Let us suppose  $X_i$  to be an important factor, thus we expect a small value of  $V_{X_{-i}}(Y|X_i)$ . In fact, fixing  $X_i$  reduces appreciably the variance of the output leading to a small value of the  $E_{X_i}(V_{X_{-i}}(Y|X_i))$ , the expected reduced variance achieved fixing  $X_i$ .

Note now that the following variance decomposition is always true:

$$E_{X_i}(V_{X_{-i}}(Y|X_i)) + V_{X_i}(E_{X_{-i}}(Y|X_i)) = V(Y). \quad (6)$$



The first term of the Equation 6 is called the residual and the second one is the main effect. We have already shown that an influent factor is associated with a small value of  $E_{X_i}(V_{X_{-i}}(Y|X_i))$  which is the residual in a general variance decomposition formula. This implies that the main effect will be large in an influent factor. Thus, we can use  $V_{X_i}(E_{X_{-i}}(Y|X_i))$  as a sensitivity measure providing an estimate of the individual effects of the factors. Normalizing by using the unconditional output variance  $V(Y)$  we obtain the **First Order Sensitivity Indices**:

$$S_i = \frac{V(E(Y|X_i))}{V(Y)} \quad (7)$$

They represent the main effect contribution of each input factor to the variance of the output.  $S_i$  can be demonstrated to be the output variance removed when we learn the true value of a given input factor  $X_i$ . Thus, it identifies factors that lead to the greatest reduction of the output variance. When  $\sum_{i=1}^k S_i = 1$  the inputs do not interact and the model is purely additive. This means that the effect of two or more inputs on the output can be simply expressed by the sum of their single effect. When  $\sum_{i=1}^k S_i < 1$ , the interactions are part of the model and the first order sensitivity index is not more able to explain the entire variance of the output. Higher order indices have to be taken into account. For instance, the **Second Order Sensitivity Index**,  $S_{ij}$ , quantify the extra amount of variance corresponding to the interaction between inputs  $i$  and  $j$  that is not explained by their individual effects. (For more details see Saltelli (2002), Saltelli et al. (2008), Saltelli et al. (2004), and Kucherenko and Mauntz (2005)). The following relation has been demonstrated to hold:

$$1 = \sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \dots + S_{12\dots k}. \quad (8)$$

## 4 UNCERTAINTY AND SENSITIVITY ANALYSIS RESULTS

In this section, we feel the need of analysing more in depth the ratings addressing the loss an investor might suffer, to improve the assessment of the risk embedded in ABSs structure.

Without loss of generality, the investor is assumed to be informed about the structure presented in Section 2.1, so that all these features have been fixed and they have been supposed do not affect the output variance of interest when evaluating this financial instrument. According to Section 2.2, the default distribution of the pool at maturity has been assumed to follow a Normal Inverse and the default curve over time has been assumed to be modelled by the Logistic function. We have already underline how these distributions are characterised by input parameters which are unknown and have to be given by expert opinions or estimated from historical data. Moreover, in order to take into account the recoveries of the defaulted assets in the pool we need to introduce in our model some assumptions on the recovery rate and the recovery time. Under these settings, the modelling of ABSs introduces an exposure to parameter uncertainty<sup>4</sup> which comes from the uncertain inputs of both distributions<sup>5</sup> and from the assumptions on default timing and on recoveries, since the resulting rating depends heavily on their values assumed true:

- the mean ( $\mu_{cd}$ ) and the standard deviation ( $\sigma_{cd}$ ) of the Normal Inverse distribution;
- $b$ ,  $c$ , and  $t_0$  in the Logistic Function;
- the recovery rate ( $RR$ ) and the recovery lag ( $T_{RL}$ ).

Each one of these parameters has been assumed to be uniformly distributed over their respectively ranges of variation which have to be fixed at the beginning. The inputs factors and their ranges have been summarised in Table 4.

Parameter	Range
$\mu_{cd}$	[5%, 30%]
<i>Coeff.Variation</i> ( $\frac{\sigma_{cd}}{\mu_{cd}}$ )	[0.25, 1]
$b$	[0.5, 1.5]
$c$	[0.1, 0.5]
$t_0$	$[\frac{T}{3}, \frac{2T}{3}]$
$T_{RL}$	[6, 36]
RR	[5%, 50%]

Table 4: Ranges for the uncertain input factors.

Some motivation to our choice of ranges is provided as follows:

### Ranges associated with $\mu_{cd}$ and $\sigma_{cd}$

The mean and standard deviation of the default distribution are typically estimated using historical data provided by the originator of the assets (e.g. see Moody's Investor Service (2005) and Raynes and Rutledge (2003)). In our SA we will assume that the mean cumulative default

<sup>4</sup>We do not have model uncertainty.

<sup>5</sup>We remind here, that the Logistic function's parameter  $a$  is the expected cumulative default rate which has to be sampled from the Normal Inverse distribution.

rate at maturity  $T$  ( $\mu_{cd}$ ) takes values in the interval  $[5\%, 30\%]$ . This is equivalent to assume that the probability of default before  $T$  for a single asset in the pool ranges from 5% to 30%. (Recall that the mean of the Normal Inverse distribution is equal to the probability of default of an individual asset).

We make the range of the standard deviation ( $\sigma_{cd}$ ) a function of  $\mu_{cd}$  by using a range for the coefficient of variation,  $\sigma_{cd}/\mu_{cd}$ . This gives us the opportunity to assume higher standard deviation (i.e. uncertainty) for high values of the default mean than for low values of the mean, which implies that we get higher correlation in the pool for high values of the mean than for low values, see Figure 6.

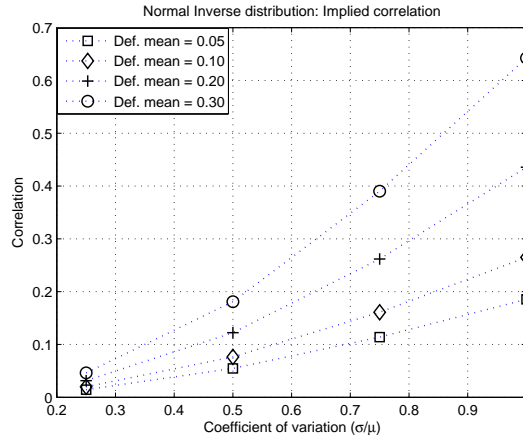


Figure 6: Implied correlation versus coefficient of variation.

### Ranges associated with $b$ , $c$ , and $t_0$ in the Logistic Function

The parameters can be estimated from empirical loss curve by fitting the Logistic curve to a historical default curve (see Raynes and Rutledge (2003)).

Because we want to cover a wide range of different default scenarios we have chosen the following parameter ranges:

- $c \in [0.1, 0.5]$ ;
- $t_0 \in [\frac{T}{3}, \frac{2T}{3}]$ ;
- $b \in [0.5, 1.5]$ .

Inspecting the behavior of the Logistic functions in Figure 4 provides some insight to possible scenarios generated with these parameter ranges and gives an intuitive understanding of the different parameters influence on the shape of the curve.

### Ranges associated with Recovery Rate and Recovery Lag

Recovery rates and recovery lags are very much dependent on the asset type in the underlying pool and the country where they are originated. For SME loans, for example, Standard and Poor's made the assumption that the recovery lag is between 12 months and 36 months depending on the country (see Standard and Poor's (2004)). Moody's uses different recovery rate ranges for

SME loans issued in, for example, Germany (25% – 65%) and Spain (30% – 50%), see Moody’s Investor Service (2009).

The range associated with the recovery lag  $T_{RL}$  has been fixed to be equal to [6, 36] months and with the recovery rate to be equal to [5%, 50%].

Since the resulting ratings depends strictly on input values used when running our model, in order to take into account that these inputs assume a discrete number of values through their ranges which have been fixed above, we evaluate ABSs model under several parameter settings. As the first, we obtain information on the uncertainty in the model having a look at the empirical distribution of the ratings over all the model evaluations.

### 4.1 Uncertainty analysis

Uncertainty analysis quantifies the uncertainty in the output due to the uncertainty in the input parameters. The inputs and their ranges create an input space of all their possible combinations. We want to explore this input space effectively in the sense of not only exploring the center of the input space but also the corner and the edges. To achieve this, the sample input values will be generated from their ranges by using Sobol’ sequences (see Section 2.2).

Under each input values combination, we evaluate the ABSs model and we provide a single rating for each tranche<sup>6</sup>. Figure 7 shows the rating empirical distribution in each one of the three notes after analysing several settings of input parameters.

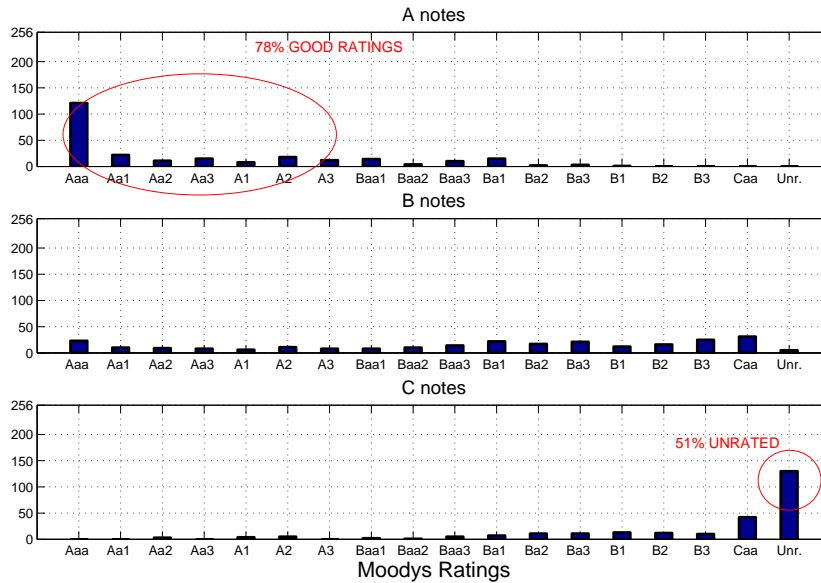


Figure 7: Moody’s Ratings empirical distribution obtained by 80 simulations.

All three histograms show evidence of dispersion in the rating outcomes. The dispersion is most significant for the mezzanine tranche resulting to be no reliable due to the oscillation

<sup>6</sup>According to Section 2, we remind that in order to get just one rating we need to run our model  $2^{14}$  times, under the same setting of parameters.

between all the ranges of variation. This analysis points out that the problem of providing a credible rating gets more difficult for the mezzanine tranche; the uncertainty is too wide and the possibility of failure in the rating determination must be reduced. The senior and the junior tranche behaves in a more stable way: we get ratings with low degree of risk at 86% of time in the A notes, and the C notes result to be unrated<sup>7</sup> 35% of time. Here we want to underline that we are not interested in getting a good rating, for example *Aaa*, for the tranches. The analysis tries to assess the reliability of the outcome understanding the dispersion of the rating distribution. As a measure of the ratings dispersion we look at the interquartile range, which is defined as the difference between the 75<sup>th</sup> percentile and the 25<sup>th</sup> percentile and it is completely independent on the quality of the ratings. We look just at the variability. Ratings percentiles are provided in Table 5. It does not come as a surprise that this difference is the highest for the B notes, 9 notches, given the very dispersed empirical distribution shown in Figure 7. From Table 5, we can also conclude that the interquartile range is equal to five and three for the A notes and the C notes, respectively.

Note	Percentile						Interquartile Range
	25	50	75	80	90	95	Number of notches
A	Aaa	Aa1	A2	A3	Baa3	Ba1	5
B	A2	Ba1	B2	B3	Caa	Caa	9
C	B2	N.R.	N.R.	N.R.	N.R.	N.R.	3

Table 5: Rating percentiles and interquartile ranges.

This dispersion in the rating distribution of course is a result of the uncertainty in the expected losses and expected average lives which underlie the ratings of each note. We have already seen in fact, how we map the expected loss and the expected average life into a rating by using the Moody's Idealised Cumulative Expected Loss Table.

Summarizing, for each single setting of input parameters we can provide a qualitative rating which is associated to two quantitative outputs, expected loss and expected average life. To get an understanding of the relationship between all of them, we can use scatter plots. Since the Figure 8, Figure 9, and Figure 10 show that there exist a positive correlation, we can conclude that the expected losses and the expected average lives are driving the rating outputs to the same direction. The 95<sup>th</sup> percentile of the expected loss and of the expected average life, for example, generates the 95<sup>th</sup> percentile of the ratings. This implies that instead of focusing just on the ratings directly, we can focus on a set of expected losses and expected average lives coming from the several input parameters settings used.

We investigate more in depth on the ABSs model and it is interesting to find out which uncertainties are driving these results. Following, we focus on the expected losses and the expected average lives with the help of sensitivity analysis techniques in order to find out which sources of uncertainty in the structure are the most responsible.

## 4.2 Sensitivity analysis

Sensitivity analysis assesses the contribution of each input parameter to the total uncertainty of the outcome. As we have already seen, our model is computationally expensive due to the

<sup>7</sup>This is not surprising because it is well known that the junior tranche is the speculative one so that the investor is supposed to be aware of C notes to be highly speculative and typically suffer significant losses due to the high uncertainty.

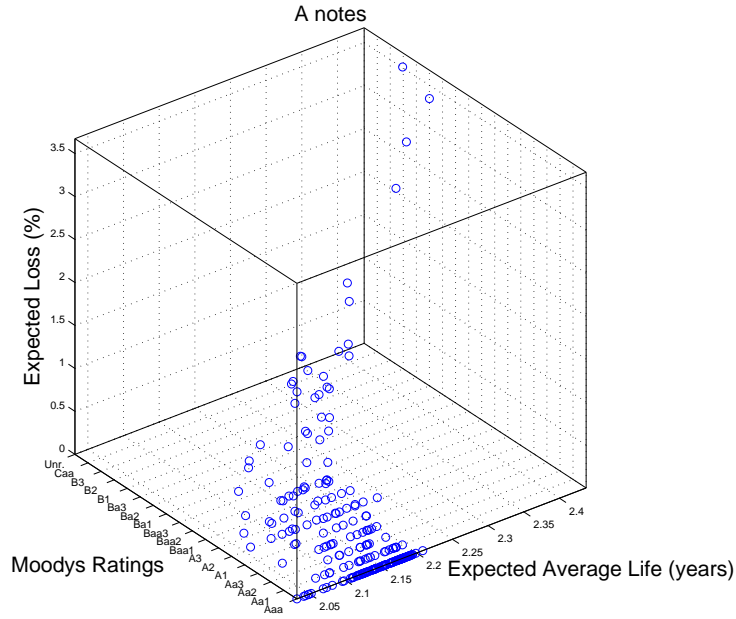


Figure 8: Expected Loss, Expected Average Life, and Ratings for the senior tranche.

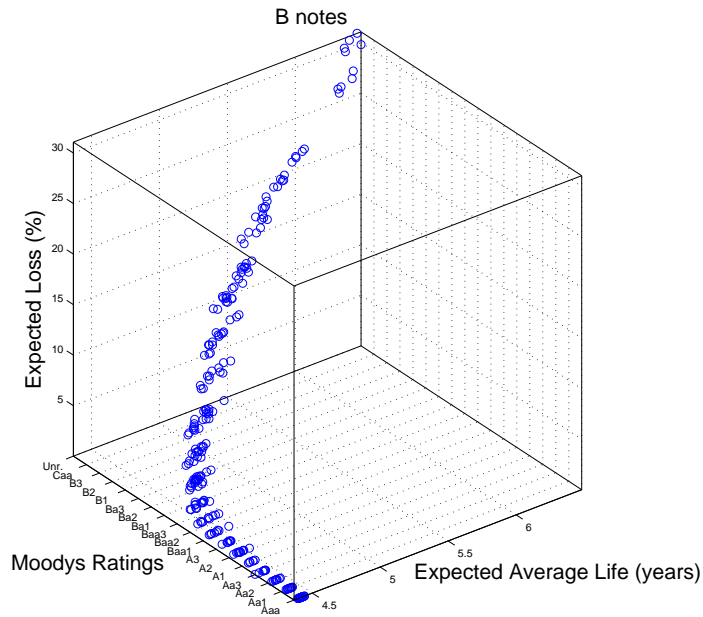


Figure 9: Expected Loss, Expected Average Life, and Ratings for the mezzanine tranche.

ABS structure's complexity so that it is wise to screen the input space in a first step. We start by using the elementary effect method; all the not influential parameters will be determined and their values will be fixed. Following, the variance based method will be applied to quantify and to distribute the uncertainty of our model among the parameters identified to be influential previously.

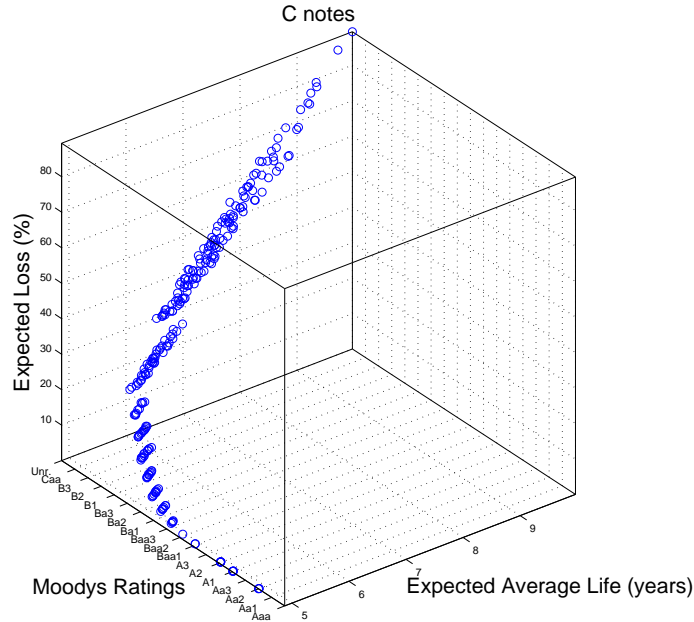


Figure 10: Expected Loss, Expected Average Life, and Ratings for the junior tranche.

The start point for both of them, is to run the model different times in order to take into account that each input can assume a different value: for each parameter setting of the input factors, we evaluate the model. The number of SA evaluations to get sensitivity analysis results depends on the technique used. We remark here that the ABS model runs  $2^{14}$  times to provide one rating under a single set of parameters.

In the elementary effect method, we select 80 settings of input parameters and we run the model for each of them<sup>8</sup>. In the variance based method, we select  $2^8$  settings of input parameters.<sup>9</sup>

#### 4.2.1 Elementary Effect

The elementary effect method provides one sensitivity measures, the  $\mu^*$ , to rank each input factor in order of importance. We have already said: higher  $\mu_i^*$  value, more important factor  $i$ . In Figure 11, Figure 12, and Figure 13 bar plots visually depict the rank of the several input factors using 80 settings of input values. The least influential factors across all outputs are the recovery lag and the Logistic functions's  $b$  parameter and hence they could be fixed without affecting the variance of the outputs of interest and therefore the ratings to a great extent. All the other parameters are influential and among them, the mean of the default distribution ( $\mu_{cd}$ ) is clearly the most important input parameter over all for all three notes. It is characterized by high  $\mu^*$  values for both the Expected Loss and the Expected Average Life of all the notes. This highlights the strong influence the mean default rate assumption has on the assessment of the ABSs. The only exception from ranking the mean default rate as the most influential input

<sup>8</sup>We apply the method with using  $r = 10$  trajectories of 4 points. Having  $k = 7$  input parameters the total number of SA model evaluations is 80 ( $N = r(k + 1)$ ). This choice has been demonstrated to produce valuable results in a general application of the sensitivity analysis.

<sup>9</sup>This choice has been demonstrated to produce valuable results in a general application of the variance based method (see Ratto and Pagano (2010 in press)).

factor is the Expected Loss of the A notes. Here the coefficient of variation is ranked the highest with the recovery rate as second and the mean default rate as third.

The input *Coef.f.Variation* and *RR* are always influential but with a lower  $\mu^*$  value than the  $\mu_{cd}$ . The  $t_0$  and  $c$  parameters are not influential in some notes but influential in some others so that they cannot be fixed without affecting the output variance of interest.

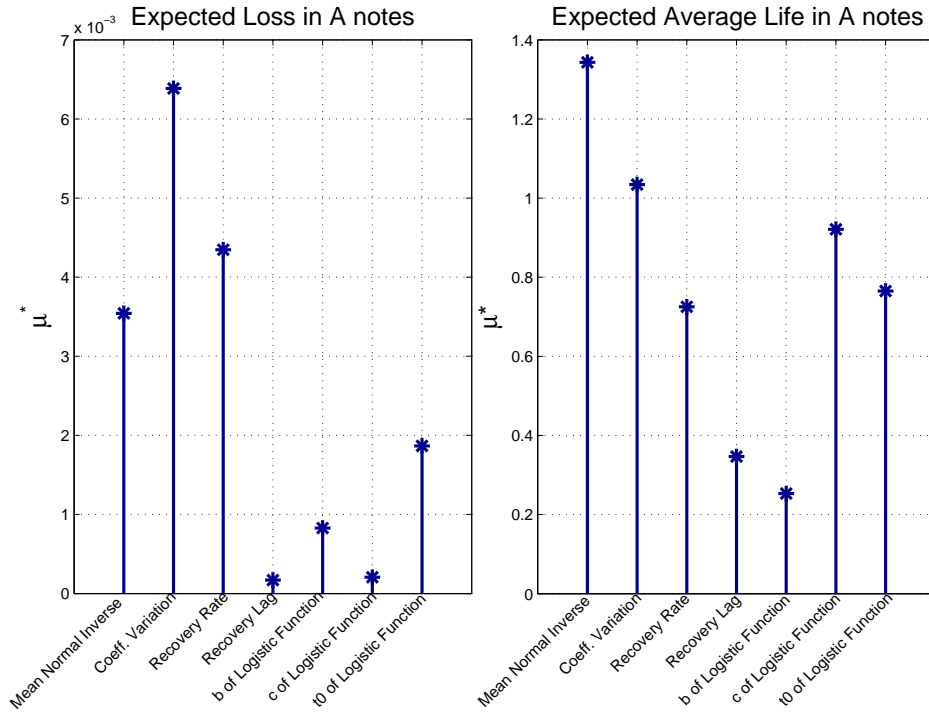


Figure 11: Bar plots of the  $\mu^*$  values for the A notes.



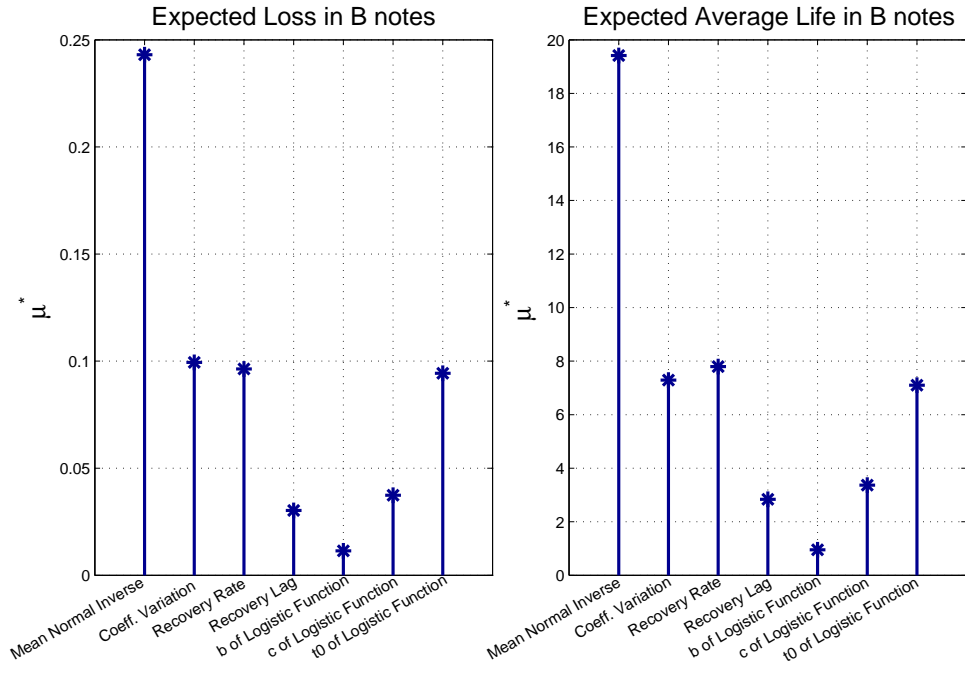


Figure 12: Bar plots of the  $\mu^*$  values for the B notes.

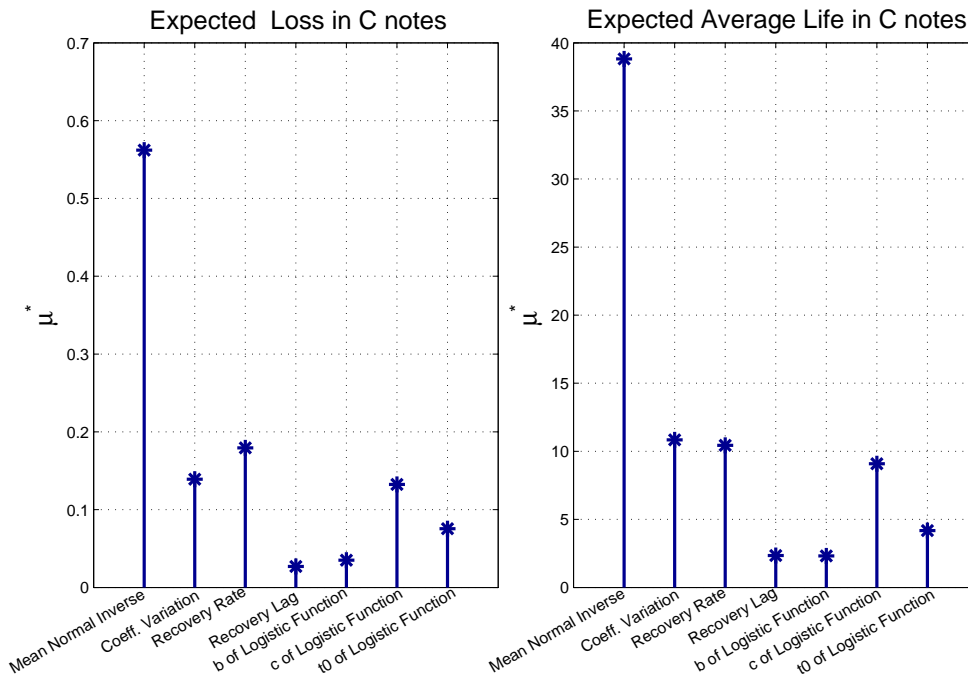


Figure 13: Bar plots of the  $\mu^*$  values for the C notes.

### Changing the thickness of junior tranche

So far, we have highlighted a problematic point concerning with the mezzanine tranche. It results to be extremely sensitive to the choice of parameters leading to the conclusion of rating to be not robust and instable. What happens improving the credit enhancement for the B notes such to decrease the probability that the holder of this tranche will lose a significant part of the investment? Do the uncertainty and sensitivity results change? Does the mezzanine tranche continue to be extremely sensitive to the choice of parameter values or are we increasing the stability and robustness of the rating?

We focus on understanding the role of the thickness for the junior tranche in affecting the rating in the mezzanine tranche. Let us suppose to increase the initial principal amount of the junior tranche keeping the principal amount for the mezzanine tranche (this leads to change the principal of the senior tranche).

Class of Note	Initial Principal Amount
A (Senior)	<b>76,000,000</b>
B (Mezzanine)	14,000,000
C (Junior)	<b>10,000,000</b>

Table 6: New structural characteristics

Figure 14 shows the empirical distribution of the ratings giving information on the uncertainty in the new structure. Unless improving the credit enhancement of mezzanine tranche, too much uncertainty affects again the B notes leading this tranche to be not reliable because of the oscillation of the ratings. The problematic point concerning with the B notes seems not to be eliminated.

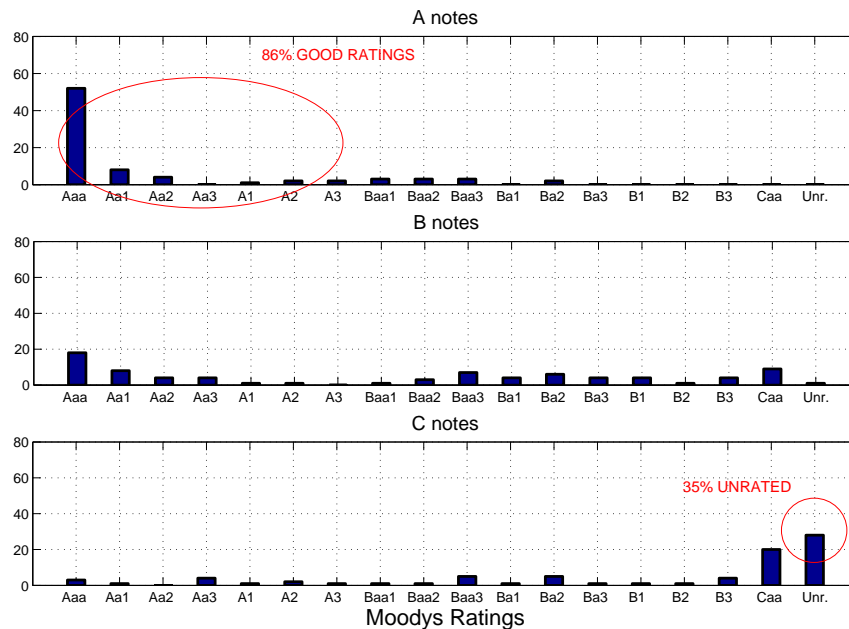


Figure 14: Moody's Ratings empirical distribution obtained by 80 simulations.

Bar plots in Figure 15, Figure 16, and Figure 17 depict the rank of the inputs accordingly to the  $\mu^*$  values for the new structure.

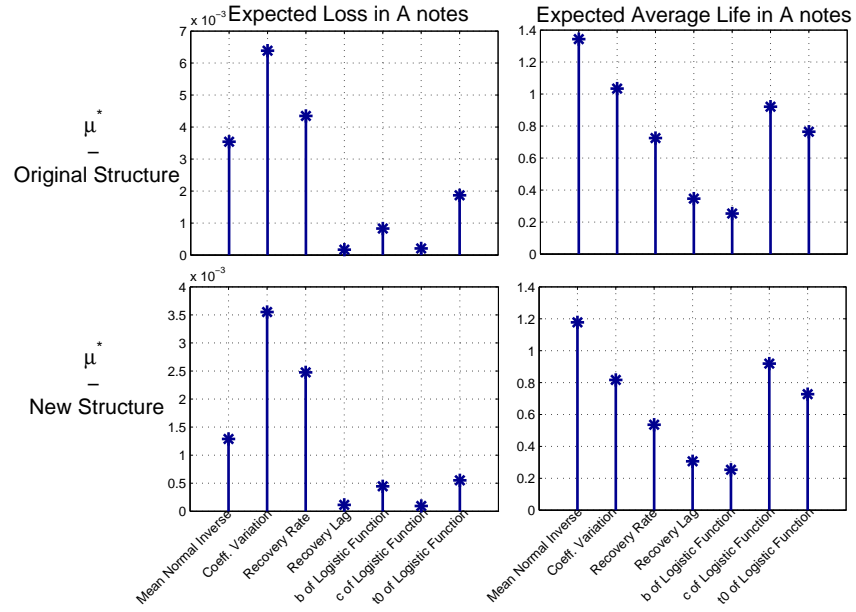


Figure 15: Bar plot of the  $\mu^*$  values for the A notes.

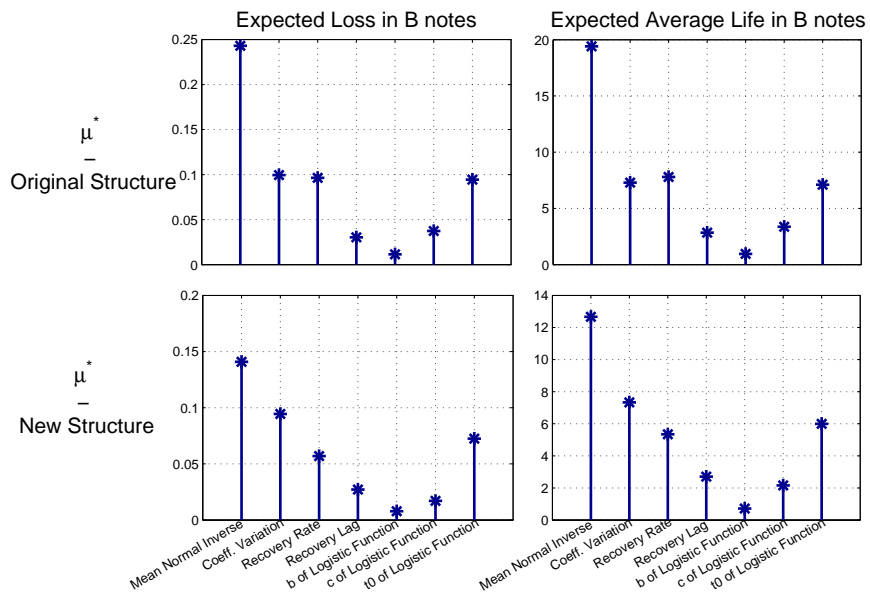


Figure 16: Bar plot of the  $\mu^*$  values for the B notes.

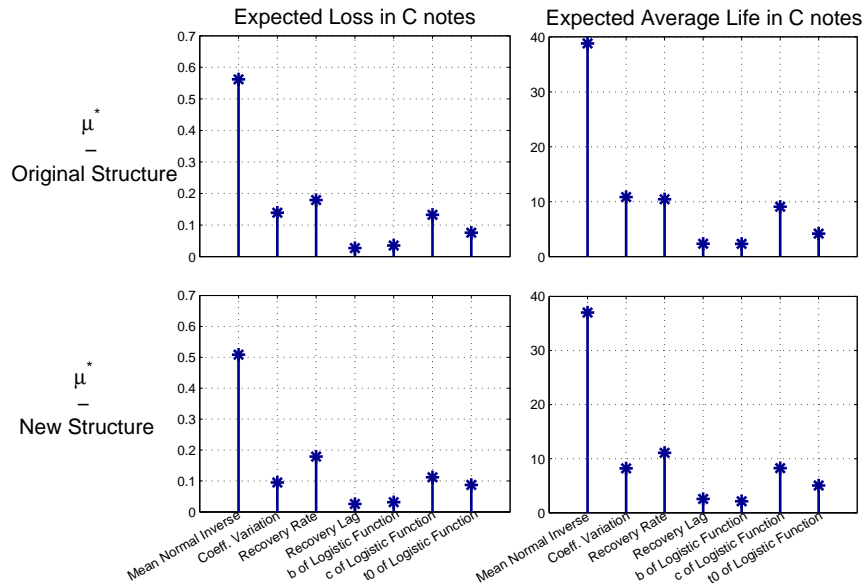


Figure 17: Bar plot of the  $\mu^*$  values for the C notes.

The SA results are consistent with that ones obtained for the original structure. Unless increasing the credit enhancement for the mezzanine tranche, its extreme sensitivity does not change. The uncertainty in the ratings of the mezzanine tranche continues to be rather high. This indicates we should improve our knowledge in order to be able to reduce the risk of failure in a rating determination.

The exploration of the inputs space by using the EE method leads to the conclusion that among all seven input factors just five of them ( $\mu_{cd}$ , *Coeff.Variation*, *RR*,  $t_0$ , and  $c$ ) play a major role in determining the uncertainty in the output rating. This leads to the need of including them in a more sophisticated analysis. We therefore proceeded to perform a more quantitative sensitivity analysis in order to assess the importance of each factor by computing its contribution to the variability of the output.

### 4.2.2 Variance Based

In the elementary effect analysis performed above, the EE method has selected five factors ( $\mu_{cd}$ , *Coef.f.Variation*, *RR*,  $t_0$ , and  $c$ ) out of seven to play a major role in determining the uncertainty in the output rating. By using variance based method we calculate the exact percentage of the output variance removed by learning the true value of these input factors taking into account the individual effect and the interactions in which each of these factors is involved.

We select now  $2^8$  settings of input parameters, we run our model for each of them and finally we obtain the first order sensitivity indices. Figure 18 depicts a clear decomposition of the output variance highlighting the main contributions due to the individual input parameters.

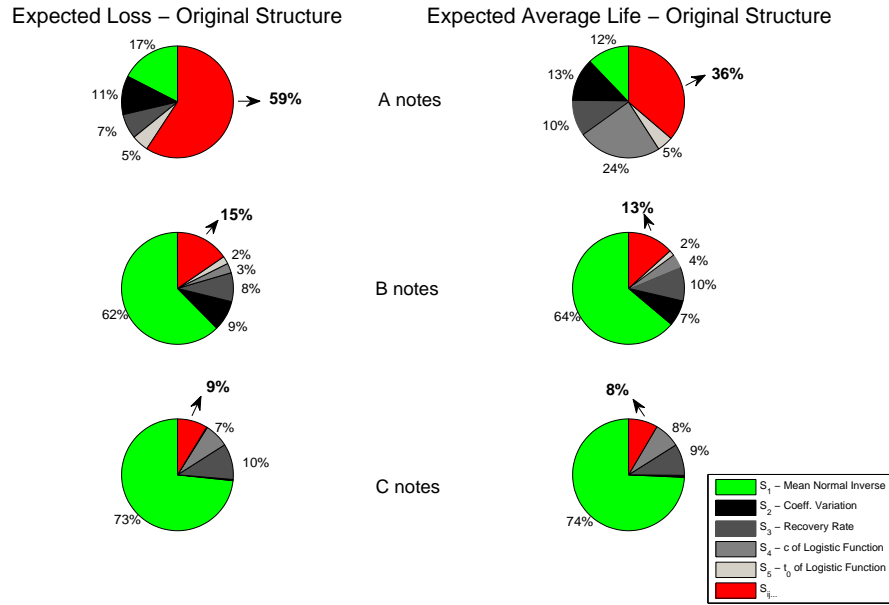


Figure 18: First order sensitivity indices for the original structure.

For the mezzanine and junior trenches the mean cumulative default,  $\mu_{cd}$ , is clearly contributing the most to the variance, accounting for approximately more than 60% and more than 70%, respectively. The uncertainty analysis performed at the begging has pointed out that the uncertainty in the mezzanine tranche is too wide leading to difficulties in a rating determination. Now, the first order sensitivity indices detect that it is possible to handling with the difficult of providing a credible results for the mezzanine tranche. Improving the knowledge of the  $\mu_{cd}$ , we can reduce the variability of the output of more than 60%. For the senior tranche the most important contributions to the output variance come from interactions between input factors indicating that the first order indices cannot solely be used to identify the most important factors and more sophisticated SA measures must be used. When interactions are involved in the model, we are not able to understand which input is the most responsible of them just using the main effect contribution. Let us have a look at the second order sensitivity index. Figure 19 depicts a new decomposition of the variance including the contributions due to the interactions between two input factors. Now, the mean cumulative default,  $\mu_{cd}$ , is clearly contributing the most to the variance in all the three notes. Less than 5% for the mezzanine and junior tranche and less than 15% in the mezzanine tranche refer to interactions among more than two factors.

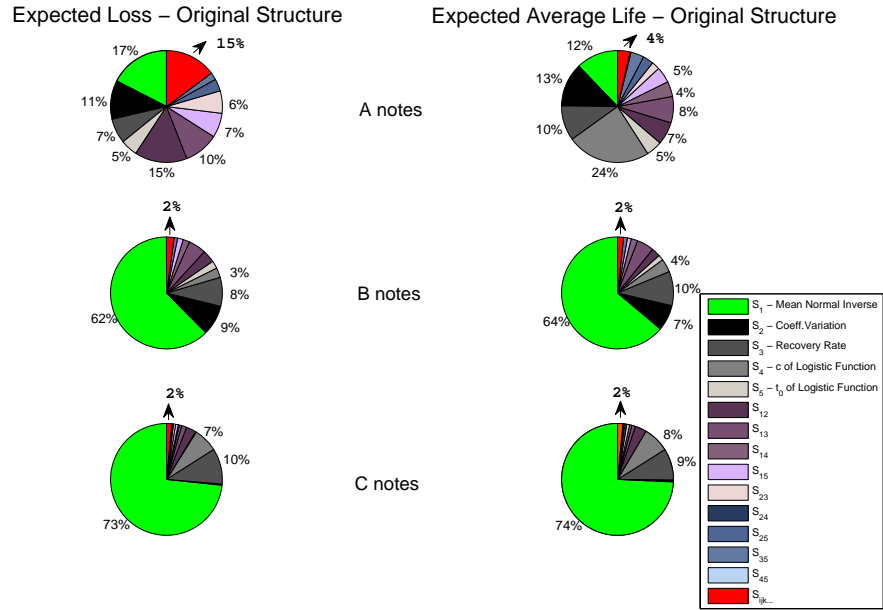


Figure 19: First and second order sensitivity indices for the original structure.

Summarising the results found out by sensitivity analysis performed above, two parameters,  $T_{RL}$  and  $b$ , were non-influential and therefore can be fixed to constant values. Among other parameters, Figure 20 shows that the most influential parameter is the mean cumulative default which is the main contributor to the uncertainty in the output with respect its main effect and the interaction effect. As this is a controllable factor, we would be encouraged to carry out further analysis searching for the optimal value of this factor in order to reduce the uncertainty in the analysis outcome. We would not need to complicate the model because we are already aware of the source of the variability in the model. According to a theoretic approach, if we would assess the true value of the mean cumulative default we could eliminate the most of the uncertainty in the model. In practice, this true value is unknown to us and it is unfeasible to find it. Unless we cannot eliminate the uncertainty in our model, at least now we are aware of it when evaluating ABSs and we know where it comes from: in particular this holds true for the mezzanine tranche. Having this in mind we find a way to live and to encompass this uncertainty in the ABSs model.

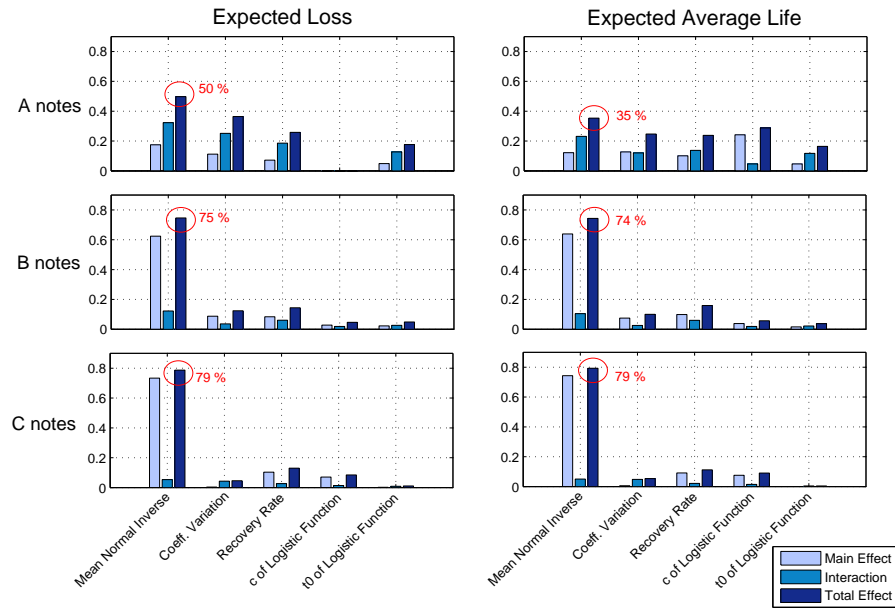


Figure 20: Sensitivity indices for the original structure.

## 5 Global Rating

In the previous sections we saw that the uncertainty in the input parameters propagates through the model and generates uncertainty in the outputs. By using sensitivity analysis we have investigated on it in order to quantify this uncertainty and identify their sources when rating ABSs. If we knew the true value of the most important inputs, we could eliminate the most of the variability in the model but in practise, these true values are unknown to us and it is unfeasible to find them. This implies that we have an intrinsic problem in the ABSs evaluation and having this in mind we try to find a way to live and encompass the uncertainty.

We propose a new rating approach that should take into account the uncertainty and should be more stable reducing the risk of cliff effects<sup>10</sup> when assigning ratings to tranches. The **global rating** is the novel strategy which assigns the ratings according to the dispersion of the credit risk giving one answer of a way forward for the rating of structure finance products.

The global rating procedure is basically the same as the one used for the uncertainty analysis and sensitivity analysis:

1. Identify the uncertain input factors, their ranges and distributions;
2. Generate several settings of input parameters in the input space;
3. For each setting run the model and give out a rating of each note;
4. Derive the global rating of each note.

### 5.1 Methodology

The main question is how to pick the global rating of the ABSs tranches. The idea is to derive it from the empirical distribution of ratings generated by several settings of input values analysed in the sensitivity analysis. The important point is that this procedure is independent of which rating methodology is used to derive the rating, this is, if it is based on expected loss or probability of default.

In order to take into account the uncertainty rather than using a single ratings which is very accurate but may easily change when changing one input value which is of course uncertain, we would rather define five global rating classes, contained one in the other, that reflect a range of possible credit risks and incorporate several underlying ratings resulting to be more stable. This global rating is given in a new scale<sup>11</sup>: *A*, *B*, *C*, *D*, and *E*. The new scale is superimposed on a rating scale used by a rating agency or by a financial institution and it is based on a percentile mapping of the underlying rating scale, that is, to assign a global rating to a tranche if a predetermined fraction of the ratings generated using the several settings of input parameters is better than or equal to a given underlying rating.

Hence, to set up the global rating scale we first have to decide on the ranges of the credit risk and of the underlying rating scale. A proposal of possible ranges for the global rating scale *A* – *E* is provided in Table 7. The global rating B in Table 7, for example, indicates that the credit risk is ranging from Low to Medium. The corresponding range in Moody’s rating scale is *Aaa* – *Baa3*. This informs the potential investor that the tranche shows low credit risk for certain scenarios but that there are scenarios where the credit risk are on a medium level.

<sup>10</sup>the risk that a small change in one or several of the input assumptions generates a dramatic change of the rating.

<sup>11</sup>This can be connected to the energy consumption scale.



Secondly, we have to choose the fraction of rating outcomes that should be laying in the credit risk range. As first attempt, under the global rating scale given in Table 7, we have defined the scale with respect to the 80th percentile of the local rating scale (in this case Moody's ratings). This mapping is shown in Figure 21. From the graph one can see that to assign a global rating *B*, for example, at least 80% of the ratings must be better than or equal to *Baa3*: the rating *Baa3* is the lowest one in the range of Moody's ratings we have obtained 80% of the times.

Global Rating	Credit Risk Range	Moody's
A	Low	A3–Aaa
B	Low to Medium	Baa3–Aaa
C	Low to High	Ba3–Aaa
D	Low to Higher	B3–Aaa
E	Low to Highest	N.R.–Aaa

Table 7: A proposal of global rating scale and the corresponding ranges in credit risk and in Moody's rating scale.

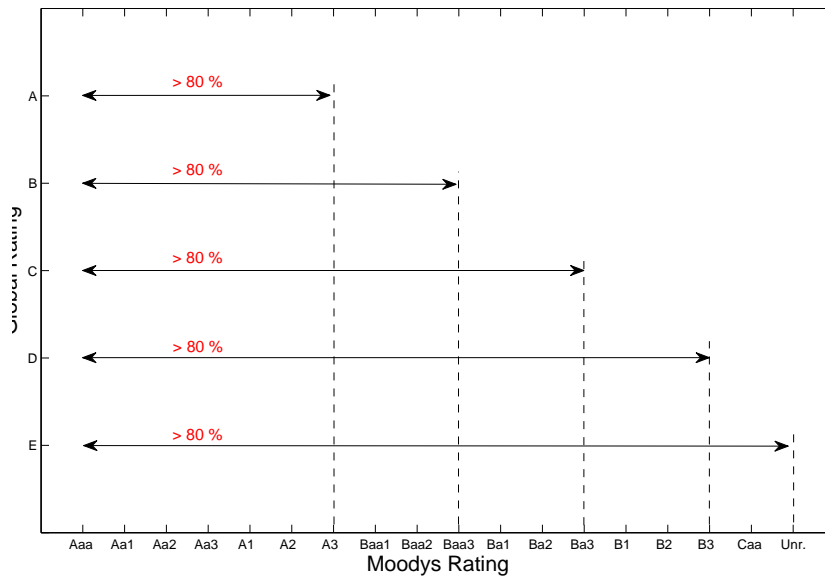


Figure 21: Example of mapping from Moody's scale to the global rating scale. The mapping is based on the 80th percentile and the percentiles for each global rating is: A: *A3*; B: *Baa3*; C: *Ba3*; D: *B3*; and E: *N.R.* (see Table 7).

Using the percentiles of the ratings in Table 5 we can derive the global ratings of the three notes. The global ratings based on the rating scale provided in Table 7 for different rating percentiles are shown in Table 8. Under the assumption to use the rating percentile equal to 80%, we obtain global ratings to be *A*, *D*, and *E* for the senior, mezzanine and junior tranche respectively. Note that this is just a first attempt.

	<b>Percentile</b>		
<b>Note</b>	75%	80%	90%
A	A	<b>A</b>	B
B	D	<b>D</b>	E
C	E	<b>E</b>	E

Table 8: Global ratings for different percentiles.

## 6 Conclusions

The valuation of different types of asset-backed securities (ABSs) have been in focus the last years due to the enormous losses anticipated by investors and the huge amount of downgrades among structured finance products. The assessment of the risk inherent in an ABSs structure and how well this risks are mitigated is detected by the ratings. The ABSs evaluation with the rating process is based on mathematical models containing a number of input variables whose values are affected by uncertainty. The uncertainty in these variables propagates through the model and produces an uncertainty in the ratings determination. The sensitivity of the rating output with respect to input assumptions has become a major concern nowadays.

We focus on a large homogeneous pool of assets backing three classes of notes (senior, mezzanine, and junior). The assets are amortizing and are assumed to be defaultable. To model the defaults in the pool we have used the Normal Inverse distribution to describe the distribution of the cumulative pool default rate at maturity. The default curve, describing the cumulative default rate's evolution over time, has been generated by the Logistic function. The uncertain input factors in the study are the mean and standard deviation of the default distribution; three of the Logistic function's parameters; and the recovery rate and the recovery lag.

The goals of this paper are two fold. Firstly, we enhance the understanding of the variability of the ratings due to the uncertainty in the input parameters. We start working with uncertainty analysis techniques to better assess and quantify the variability in the structured finance product output due to the variability in the inputs. This analysis points out that the empirical distribution for the ratings results to be dispersed. The problem of providing a credible rating gets more difficult for the mezzanine tranche; the uncertainty is too wide and the possibility of failure in the rating determination must be reduced. Due to this result, we advocate the use of global sensitivity analysis to understand the main sources of output uncertainties and how the uncertainty in the output can be allocated to the different sources of uncertainty in the inputs. We quantify the percentage of output variance that each input factor is accounting for and we also detect how interactions among input parameters affect the rating variability exploring the whole input space. We answer to the following questions: Is the rating of an ABS reliable? Where does the uncertainty come from, i.e. which input factors are more important in determining the uncertainty in the rating response? Can I quantify the exact percentage of the variability in the output that can be allocated to each input? We figure out that among all input parameters, the Logistic function's  $b$  and the  $T_{RL}$  are not influential at all. The all other inputs are influential and they cannot be fixed without affecting the output variance. Among these, the most influential one is the mean cumulative default which is the main contributor to the uncertainty in the output with respect to its main effect and to the interaction effect. For the junior and mezzanine tranches, the main effect (without the interactions) of the mean cumulative default contributes the most to the output variance accounting for approximately more than 70% and 60% respectively. In the senior tranche the interactions play an important role but anyway the mean cumulative default keeps to be the most influent parameters taking into account the contribution of its main effect and its interaction with the other inputs. According to a theoretic approach, if we would assess the true value of the mean cumulative default we could eliminated the most of uncertainty in the model. In practice, this true value is unknown to us and it is unfeasible to find it. Unless we cannot eliminate the uncertainty in our model, at least now we are aware of it when evaluating ABSs and we know where it comes from: in particular this holds true for the mezzanine tranche where the empirical distribution of the ratings result to be too much dispersed. Having this in mind we propose a way to live and to encompass this uncertainty in the ABSs model.

The second goal of the paper in fact is to work out a novel rating approach for the asset backed securities called *global rating*, that takes this uncertainty in the output into account when assigning ratings to tranches. In the global rating a new rating scale based on percentile is used that indicates the range of the credit risk of an asset backed security. The global ratings should therefore become more stable and reduce the risk of cliff effects, that is, that a small change in one or several of the input assumptions generates a dramatic change of the rating. To set up the global rating scale we first have decided on the ranges of the credit risk and of the underlying rating scale. Secondly, we have chosen the fraction of rating outcomes that should be laying in the credit risk range so that we have defined the scale with respect to the 80th percentile of the local rating scale (in this case Moody's ratings). Under the assumptions that are just a first attempt, we obtain global ratings to be *A*, *D*, and *E* for the senior, mezzanine and junior tranche respectively.

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